Measuring the Density Fluctuation From the Cluster Gas Mass Function

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ABSTRACT

We investigate the gas mass function of clusters of galaxies to measure the density fluctuation spectrum on cluster scales. The baryon abundance confined in rich clusters is computed from the gas mass function and compared with the mean baryon density in the universe which is predicted by the Big Bang Nucleosynthesis. This baryon fraction and the slope of the gas mass function put constraints on σ_8, the rms linear fluctuation on scales of $8h^{-1}$Mpc, and the slope of the fluctuation spectrum, where h is the Hubble constant in units of 100 km s^{-1}Mpc^{-1}. We find $\sigma_8 = 0.80 \pm 0.15$ and $n \sim -1.5$ for $0.5 \leq h \leq 0.8$, where we assume that the density spectrum is approximated by a power law on cluster scales: $\sigma(r) \propto r^{-\frac{3+n}{2}}$. Our value of σ_8 is independent of the density parameter, Ω_0, and thus we can estimate Ω_0 by combining σ_8 obtained in this study with those from Ω_0-dependent analyses to date. We find that $\sigma_8(\Omega_0)$ derived from the cluster abundance such as the temperature function gives $\Omega_0 \sim 0.5$ while $\sigma_8(\Omega_0)$ measured from the peculiar velocity field of galaxies gives $\Omega_0 \sim 0.2 - 1$, depending on the technique used to analyze peculiar velocity data. Constraints are also derived for open, spatially flat, and tilted Cold Dark Matter models and for Cold + Hot Dark Matter models.

Subject headings: cosmology: observations — galaxies: clusters: general

1. Introduction

To measure the spectrum of density fluctuations is one of the key issues in discussing the structure formation in the universe. Clusters of galaxies are suitable objects to measure the spectrum on scales of $\sim 10h^{-1}$ Mpc, where h is the Hubble constant in units of 100 km s^{-1}Mpc^{-1}. This is because the abundance of clusters is sensitive to the nature of the spectrum, in particular the amplitude, and because the fluctuations on cluster scales can be reliably discussed by linear theory.

Henry & Arnaud (1991) derived $\sigma_8 = 0.59 \pm 0.02$ and $n = -1.7^{-0.65}_{+0.35}$ from the X-ray temperature function of clusters for $\Omega_0 = 1$ universes, where σ_8 is the rms linear fluctuation on scales of $8h^{-1}$Mpc, Ω_0 is the cosmological density parameter, and they assumed that the density spectrum is approximated by a power law in wavenumber as $P(k) \propto k^n$ on cluster scales. White, Efstathiou, & Frenk (1993a) obtained $\sigma_8 \simeq (0.57 \pm 0.05)\Omega_0^{-0.56}$ using the spatial number density of rich clusters. Similar results for σ_8 were also obtained by other authors for open and spatially flat Cold Dark Matter (CDM) models using Henry & Arnaud's (1991) temperature function data; Eke, Cole, & Frenk (1996) found $\sigma_8 = (0.50\pm0.04)\Omega_0^{-\alpha}$ and Viana & Liddle (1996) found $\sigma_8 = 0.6\Omega_0^{-\alpha}$; in both estimates α varies from ~ 0.4 to ~ 0.6 depending on the dark matter model assumed. There is, however, a problem with those measurements of σ_8 that one cannot know σ_8 unless Ω_0 is given; in general, the measured quantity is not σ_8 but a combination of σ_8 and Ω_0 like $\sigma_8\Omega_0^{0.6}$. This problem is common to almost all methods for measuring σ_8, such as the one using the peculiar velocity field of galaxies.

In this paper, we measure observationally the fraction of baryons confined in clusters of galaxies to the total baryons in the universe using the cluster gas mass function, and give Ω_0-independent measurements of σ_8 by comparing the observed baryon fraction with the theoretical prediction. As will be seen in § 3, the theoretical derivation of the baryon fraction does not depend on Ω_0, since the baryon fraction measures essentially the fraction of density fluctuations with an overdensity larger than the critical value. We find $\sigma_8 = 0.80\pm0.15$, with the quoted errors including the uncertainties in h ($0.5 \leq h \leq 0.8$). The power-law index of the spectrum, n, is also measured.

The plan of this paper is as follows. In § 2 we derive the cluster gas mass function from the observed sample and compute the fraction of baryons confined in rich clusters. In § 3, we describe an analytic model of gravitational halo formation to predict the fraction of mass confined in dark haloes with a given mass range. In § 4, we estimate σ_8 and n. We also derive constraints on three sets of CDM models and a set of Cold + Hot Dark Matter (CHDM) models in § 4. Finally, § 5 summarizes our conclusions.

2. Data

In this section, we first derive the gas mass function of clusters from X-ray data, and then estimate the fraction of baryons confined in clusters with a given gas mass range. Ebeling et al. (1996a) constructed an X-ray luminosity function (XLF) of clusters of galaxies from the *ROSAT* Brightest Cluster Sample, which is an X-ray selected, flux limited sample of 172 clusters compiled from *ROSAT* All-Sky Survey data. They found that the XLF is well fitted by a Schechter function:

$$\phi(L)dL = AL^{-\alpha} \exp\left(-\frac{L}{L_\star}\right) dL \qquad (1)$$

with $A = (4.58 \pm 0.76) \times 10^{-6}~h^3~\mathrm{Mpc}^{-3} \times (0.25 \times 10^{44}~h^{-2}~\mathrm{erg~s}^{-1})^{\alpha-1}$, $L_\star = (2.23\pm0.42)\times10^{44}~h^{-2}~\mathrm{erg~s}^{-1}$, and $\alpha = 1.78 \pm 0.09$. Here, L is the X-ray luminosity in the $0.1 - 2.4$ keV band.

Ebeling et al. (1996b) presented an X-ray sample of 242 Abell clusters from *ROSAT* All-Sky Survey data in which X-ray luminosity in the $0.1 - 2.4$ keV band is tabulated. Among those, the gas mass within the Abell (1958) radius ($\equiv 1.5h^{-1}$ Mpc) is given for 40 clusters in Jones & Forman (1984) and Arnaud et al. (1992). The Abell radius is close to the typical virialization radius of rich clusters, and thus it is reasonable to suppose that most of the cluster gas is within the Abell radius. From the luminosity and the gas mass data of the forty clusters, we find

$$M_{\mathrm{gas}} = (0.34\pm0.04)$$
$$\times \left(\frac{L}{10^{44}}\right)^{0.65\pm0.07} (h^{-2.5}10^{14}M_\odot), \qquad (2)$$

where L is in units of h^{-2} erg s^{-1} and M_{gas} is the gas mass within the Abell radius. When estimating the errors in the coefficients of the L-M_{gas} relation (eq.[2]), we assume that the observed M_{gas} for each cluster has an error of $\sigma_{\log M_{\mathrm{gas}}} = 0.25$, which is the typical scatter of the data points around equation (2).

Figure 1 shows the distribution of the forty clusters on the L - M_{gas} plane.

From the XLF and the L - M_{gas} relation, we obtain the gas mass function:

$$\psi(M_{\text{gas}})dM_{\text{gas}}$$

$$= (1.28 \pm 0.46) \times 10^{-6} \times \left(\frac{M_{\text{gas}}}{M_{\star}} \right)^{-2.20 \pm 0.15}$$

$$\times \exp \left[-\left(\frac{M_{\text{gas}}}{M_{\star}} \right)^{1.54 \pm 0.02} \right] d \left(\frac{M_{\text{gas}}}{M_{\star}} \right) (h^3 \text{ Mpc}^{-3}), \tag{3}$$

where $M_{\star} = 0.57 \times 10^{14} h^{-2.5} M_{\odot}$. This gas mass function is consistent with the (cumulative) gas mass function derived by Burns et al. (1996) using optically selected poor and rich clusters.

We now compute the fraction of baryons confined in rich clusters. This calculation consists of three steps: (1) derive the gas density in the universe contributed from clusters, $\rho_{\text{cl}}^{\text{gas}}$, from equation (3); (2) add to $\rho_{\text{cl}}^{\text{gas}}$ the contribution from the stellar mass of cluster galaxies to obtain the baryon density contributed from clusters, $\rho_{\text{cl}}^{\text{B}}$; (3) compute the baryon fraction $f_{\text{B}} \equiv \rho_{\text{cl}}^{\text{B}}/\rho_0^{\text{B}}$, where ρ_0^{B} is the mean baryon density in the universe predicted by the Big Bang Nucleosynthesis.

First, we compute $\rho_{\text{cl}}^{\text{gas}}$ by

$$\rho_{\text{cl}}^{\text{gas}} = \int_{M_1}^{M_2} M \psi(M) dM, \tag{4}$$

where M_1 and M_2 are the lower and upper limits of cluster gas mass. Since the XLF and L - M_{gas} relation are available over $10^{42} \lesssim L \lesssim 10^{45}$ and $10^{43} \lesssim L \lesssim 10^{45}$ h^{-2} erg s^{-1}, respectively, we set $(M_1, M_2) = (7.6 \times 10^{12}, 1.5 \times 10^{14})$ $(h^{-2.5} M_{\odot})$, which corresponds to $(L_1, L_2) = (10^{43}, 10^{45})$ $(h^{-2}$ erg s$^{-1})$. Performing the integration in equation (4) [1], we obtain

$$\rho_{\text{cl}}^{\text{gas}} = (1.51 \pm 0.32) \times 10^8 \ h^{0.5} \ M_{\odot} \text{ Mpc}^{-3}, \tag{5}$$

which is translated into the density parameter $\Omega_{\text{cl}}^{\text{gas}} = (5.45 \pm 1.17) \times 10^{-4} \ h^{-1.5}$. This value of $\Omega_{\text{cl}}^{\text{gas}}$ agrees roughly with what Persic & Salucci (1992) obtained for a somewhat narrower mass range of clusters, $\Omega_{\text{cl}}^{\text{gas}} \simeq 3.5 \times 10^{-4} \ h^{-1.3}$, from an analysis using another XLF derived before *ROSAT*.

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[1] Actually, we calculate $\int_{L_1}^{L_2} M(L)\phi(L)dL$ in order to evaluate the errors in $\rho_{\text{cl}}^{\text{gas}}$ more easily.

We fit the observed gas mass function by a power law over the range of M_1 and M_2 to find

$$\psi(M) \propto M^{-3.3 \pm 0.2}. \tag{6}$$

This power-law index will also be used to put constraints on the density fluctuation spectrum, in particular its slope.

Second, we add the contribution from the stellar mass of galaxies in clusters to $\rho_{\text{cl}}^{\text{gas}}$. According to White et al. (1993b), the mass ratio of the stellar component to the hot gas is $0.2h^{1.5}$ for the Coma Cluster. Since Coma is a typical rich cluster, we assume that other clusters have the same ratio. Under this assumption, the density of baryons in clusters, $\rho_{\text{cl}}^{\text{B}}$, is calculated by

$$\rho_{\text{cl}}^{\text{B}} = \rho_{\text{cl}}^{\text{gas}}(1 + 0.2h^{1.5}). \tag{7}$$

Finally, let us derive f_{B}. The density parameter of baryons in the universe, Ω_0^{B}, is estimated using the Big Bang Nucleosynthesis. We take Walker et al.'s (1991) estimate: $\Omega_0^{\text{B}} h^2 = 0.0125 \pm 0.0025$. We do not adopt recent estimates based on the deuterium abundance in QSO absorption systems, because they are still quite controversial (*e.g.*, Carswell et al. 1996; Rugers & Hogan 1996; Tytler, Fan, & Burles 1996; Wampler 1996). Since $\rho_0^{\text{B}} = \rho_0^{\text{cr}}\Omega_0^{\text{B}}$ ($\rho_0^{\text{cr}} \equiv 3H_0^2/8\pi G$ is the critical matter density and H_0 is the Hubble constant), we obtain

$$f_{\text{B}} = (0.044 \pm 0.013)(1 + 0.2h^{1.5})h^{0.5}. \tag{8}$$

The errors are at 1σ levels and include both the errors in equation (5) and in Ω_0^{B}.

3. Model

In this section we derive analytically the fraction of matter confined in haloes with a given mass range, based on the gravitational halo formation model. The Press-Schechter (1974) theory predicts the number density of haloes with a comoving radius r as

$$n(r)dr = -\frac{3}{4\pi} \left(\frac{2}{\pi} \right)^{\frac{1}{2}} \delta_{\text{c}}$$

$$\times \frac{1}{r^3} \frac{1}{\sigma^2(r)} \frac{d\sigma(r)}{dr} \exp \left[-\frac{\delta_{\text{c}}^2}{2\sigma^2(r)} \right] dr, \tag{9}$$

where δ_{c} is the critical linear overdensity and $\sigma(r)$ is the rms of the linear density fluctuations in top-hat windows of radius r; σ_8 is defined as $\sigma(8h^{-1}\text{Mpc})$.

The fraction of matter in the universe which is confined in haloes with $r_1 \leq r \leq r_2$ is expressed as

$$f_\mathrm{M} = \frac{\int_{r_1}^{r_2} M(r) n(r) dr}{\rho_0}. \qquad (10)$$

Here, $M(r)$ is the total mass of the halo with a comoving radius r and $\rho_0 \equiv \rho_0^\mathrm{cr} \Omega_0$. We assume that the total mass of haloes is within the Abell radius. Since

$$M(r) = \frac{4\pi}{3} \rho_0 r^3, \qquad (11)$$

equation (10) is rewritten as

$$f_\mathrm{M} = \int_{r_1}^{r_2} \frac{4\pi}{3} r^3 n(r) dr. \qquad (12)$$

We then assume that the baryon-to-total mass ratio of haloes is equal to ρ_0^B/ρ_0. This assumption is reasonable because N-body simulations to date have suggested that hydrodynamical effects do not significantly influence the baryon-to-total mass ratio within the Abell radius (*e.g.*, White et al. 1993b; Lubin et al. 1996 and references therein). Thus, a cluster with a baryon mass M_B have a total mass $M_\mathrm{tot} = M_\mathrm{B} \rho_0 / \rho_0^\mathrm{B}$, and thus have a comoving radius:

$$r = \left(\frac{3 M_\mathrm{B}}{4 \pi \rho_0^\mathrm{B}} \right)^{\frac{1}{3}}. \qquad (13)$$

Equations (9), (12), and (13) show that f_M, which is equal to f_B by the assumption above, is computed if $\sigma(r)$, ρ_0^B, and the lower and upper limits of M_B are given. We take $\delta_\mathrm{c} = 1.69$ throughout this paper for simplicity, because δ_c depends on Ω_0 and the cosmological constant λ_0 only weakly (*e.g.*, Lilje 1992; Lacey & Cole 1993).

4. Results and Discussion

4.1. Estimates of the Amplitude and Shape of the Fluctuation

In this subsection, we assume that the density fluctuation spectrum is a power law on cluster scales:

$$\sigma(r) = \sigma_8 \left(\frac{r}{8 h^{-1} \mathrm{Mpc}} \right)^{-\frac{3+n}{2}}, \qquad (14)$$

and estimate σ_8 and n from f_B and the slope of the gas mass function.

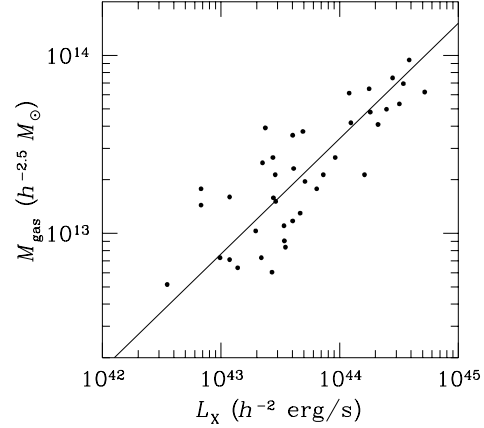Figure 2 shows the constraints on σ_8 and n. Results are shown for two extreme values of h, since the

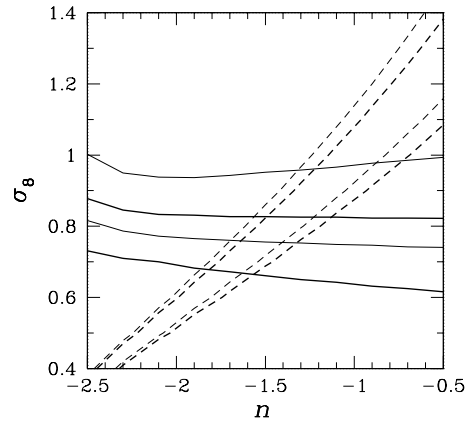Fig. 1.— L - M_gas relation of forty clusters. Equation (2) is shown as the solid line.

Fig. 2.— Constraints on σ_8 and n. The region between the thick (thin) solid lines indicates the range allowed by the baryon fraction f_B for $h = 0.5$ (0.8). The region between the thick (thin) dashed lines is for the constraint for $h = 0.5$ (0.8) derived from the slope of the gas mass function.

4

observed and predicted baryon fractions depend on h (see eqs.[8] and [13]). The two values of h taken here, 0.5 and 0.8, roughly correspond to the lower and upper limits of recent measurements, respectively (see, *e.g.,* Freedman 1996). The region between the two thick (thin) solid lines indicates the range allowed by the baryon fraction, f_B, for $h = 0.5$ (0.8); for each h, the region corresponds to the 1σ range of f_B (eq.[8]). The region between the two thick (thin) dashed lines is for the constraint from the slope of the gas mass function for $h = 0.5$ (0.8); for each h, the region corresponds to the 1σ range of the slope (eq.[6]). In deriving the constraint from the slope of the gas mass function, we fit the theoretical mass function with a power law, $n(M) \propto M^a$, over the range of M corresponding to the gas mass range (M_1 and M_2 in § 3), and compare a with the power-law index of equation (6). We see in Figure 2 two features common to the $h = 0.5$ and 0.8 cases. First, f_B places almost the unique constraint on σ_8 irrespective of n. This is because 8 h^{-1}Mpc is the typical comoving radius of rich clusters. Second, a higher σ_8 is required for a higher n to reproduce the observed slope of the gas mass function.

From Figure 2 we find the allowed values of σ_8 and n to be $(\sigma_8, n) = (0.75 \pm 0.1, -1.5 \pm 0.4)$ for $h = 0.5$, and $(0.85 \pm 0.1, -1.3 \pm 0.4)$ for $h = 0.8$; a lower h gives a lower σ_8. Adopting $0.5 \leq h \leq 0.8$ as the observational uncertainty in h, we can express the range of σ_8 by

$$\sigma_8 = 0.80 \pm 0.15. \qquad (15)$$

In the rest of this subsection, we compare σ_8 obtained above with those from Ω_0-dependent analyses to date, and then estimate Ω_0. Henry & Arnaud (1991) derived $\sigma_8 = 0.59 \pm 0.02$ and $n = -1.7^{-0.65}_{+0.35}$ from the cluster temperature function assuming $\Omega_0 = 1$. The value of n they obtained agrees very well with what we obtain above. White et al. (1993a) derived $\sigma_8 \simeq (0.57 \pm 0.05)\Omega_0^{-0.56}$ using the cumulative abundance of rich clusters. The basic procedures by Henry & Arnaud (1991) and White et al. (1993a) to measure σ_8 are the same in the sense that both of them used the cluster abundance and applied the Press-Schechter (1974) theory. Thus, White et al.'s (1993a) estimate of σ_8 for arbitrary Ω_0 may be regarded as a generalization of Henry & Arnaud's (1991). Another method for measuring σ_8 is to use the peculiar velocity field of galaxies, though a large inconsistency is seen among recent measurements. Two rep-

resentative measurements are those given in Kolatt & Dekel (1997) and Willick et al. (1996). Kolatt & Dekel (1997) obtained $\sigma_8 = (0.71 - 0.77[\pm 0.12])\Omega_0^{-0.6}$ from the POTENT analysis (Dekel, Bertschinger, & Faber 1990; Dekel 1994) of the Mark III peculiar velocity catalog (Willick et al. 1997). Willick et al. (1996) found $\sigma_8 = (0.34 \pm 0.05)\Omega_0^{-0.6}$, applying another technique to the same catalog. One can also measure σ_8 (as a function of Ω_0) using the redshift-space anisotropy of galaxy clustering; examples are Fisher et al. (1994) and Cole, Fisher, & Weinberg (1995), both of which derived similar values to what Willick et al. (1996) obtained [2].

Since our estimate of σ_8 is independent of Ω_0 while those given in the previous papers listed above depend on Ω_0 in a way like $\sigma_8(\Omega_0) = \mathrm{const.} \times \Omega_0^{-0.6}$, we can compute Ω_0 by comparing them. We take three estimates of $\sigma_8(\Omega_0)$ as representatives of the previous ones: those given in White et al. (1993a), Kolatt & Dekel (1997), and Willick et al. (1996). A comparison between our σ_8 and White et al.'s (1993a) $\sigma_8(\Omega_0)$ gives $\Omega_0 = 0.6^{+0.3}_{-0.2}$ (for $h = 0.5$) and $\Omega_0 = 0.5^{+0.2}_{-0.2}$ ($h = 0.8$). From Kolatt & Dekel's (1997) measurement we obtain $\Omega_0 = 1.0^{+0.7}_{-0.5}$ ($h = 0.5$) and $\Omega_0 = 0.8^{+0.5}_{-0.3}$ ($h = 0.8$). Finally, Willick et al.'s (1996) $\sigma_8(\Omega_0)$ gives $\Omega_0 = 0.3^{+0.2}_{-0.1}$ ($h = 0.5$) and $\Omega_0 = 0.2^{+0.1}_{-0.1}$ ($h = 0.8$). We then compare these values with that computed from the cluster baryon-to-total mass ratio and Ω_0^B. White et al. (1993b) found that the baryon-to-total mass ratio for the Coma Cluster is $0.009 + 0.05h^{-1.5}$ (the 1σ error at a given value of h is $\sim 30\%$). Adopting this ratio as the global value in the universe and using $\Omega_0^B = (0.0125 \pm 0.0025)h^{-2}$, we obtain $\Omega_0 = 0.33 \pm 0.1$ for $h = 0.5$ and $\Omega_0 = 0.25 \pm 0.1$ for $h = 0.8$. These values of Ω_0 are in very good agreement with those derived from Willick et al.'s (1996) measurement of $\sigma_8(\Omega_0)$, but are smaller than those derived from White et al.'s (1993a) and Kolatt & Dekel's (1997), though the disagreements are less than 2σ levels. To summarize, we find that there exists a region on the Ω_0 - σ_8 plane where the three constraints overlap with each other within the observational uncertainties, *i.e.*, the constraints on σ_8 obtained from the gas mass function in this paper,

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[2]The quantity Fisher et al. (1994) and Cole et al. (1995) measured is not σ_8 but $\beta_I \equiv \Omega_0^{0.6}/b_I$, where b_I is the bias parameter of $IRAS$ galaxies. However, as is shown in Willick et al. (1996), one can derive $\sigma_8(\Omega_0)$ from β_I and the observed rms fluctuations of $IRAS$ galaxies on an $8h^{-1}$ Mpc scale, assuming that biasing is independent of scale.

on $\sigma_8(\Omega_0)$ derived from the Ω_0-dependent analyses discussed above, and on Ω_0 derived from the cluster baryon-to-total mass ratio and Ω_0^B.

4.2. Constraints on Cosmological Models

In this subsection, we examine four sets of cosmic structure formation models including (1) open ($\lambda_0 = 0$) CDM models with Ω_0 the free parameter, (2) spatially flat CDM models with Ω_0 the free parameter, (3) tilted CDM models with n the free parameter, and (4) CHDM models with the density parameter of the hot dark matter, Ω_ν, the free parameter. In what follows, n denotes the power-law index of the primordial (*i.e.*, before filtered by the transfer function) density fluctuation spectrum, not the index of the current spectrum on cluster scales. Table 1 summarizes the model parameters. For each set, we constrain the above free parameter by requiring that it matches simultaneously the observed baryon fraction and the 4-year *COBE* DMR data (Bennett et al. 1996). We take the functional forms of the CDM and CHDM spectra from those given in Kitayama & Suto (1996) and Ma (1996), respectively (these authors took the form of the CDM spectrum from Bardeen et al. 1986 and corrected it for the baryon component). We do not use the slope of the gas mass function to constrain the models because the CDM and CHDM spectra on cluster scales give power-law indices which are close to those derived in § 4.1. We assume $\Omega_0 = 1$ for the tilted CDM models and the CHDM models for definiteness, although this assumption is apparently inconsistent with the values of Ω_0 derived from Ω_0^B and the baryon-to-total mass ratio for Coma (see § 4.1).

Open CDM models

Figure 3 plots the constraints on σ_8 and Ω_0 for open CDM models. Results are shown for two extreme values of h. The region between the two thick (thin) solid lines is allowed by the baryon fraction, f_B, for $h = 0.5$ (0.8). The thick (thin) dashed line indicates σ_8 for $h = 0.5$ (0.8) which is estimated from the

Table 1: Model parameters.

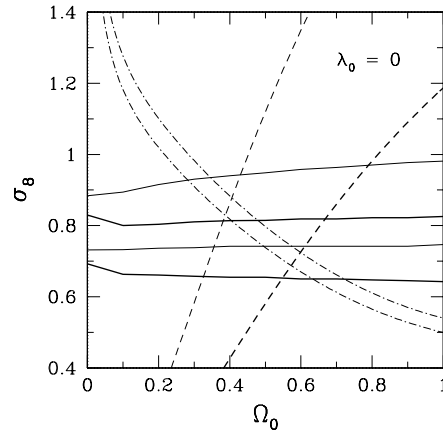| Model	h	n	Ω_0	λ_0	Ω_ν
open CDM	$0.5, 0.8$	1	varying	0	0
spatially flat CDM	$0.5, 0.8$	1	varying	$1 - \Omega_0$	0
tilted CDM	0.5	varying	1	0	0
CHDM	0.5	1	1	0	varying

Fig. 3.— Constraints on σ_8 and Ω_0 for open CDM models. The region between the thick (thin) solid lines indicates the range allowed by the baryon fraction f_B for $h = 0.5$ (0.8). The value of σ_8 given by the 4-year *COBE* data for $h = 0.5$ (0.8) is shown by the thick (thin) dashed line. The region between the dash-dotted lines indicates the constraint derived by Kitayama & Suto (1997).
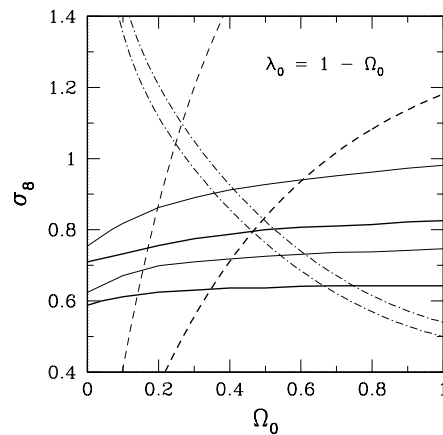

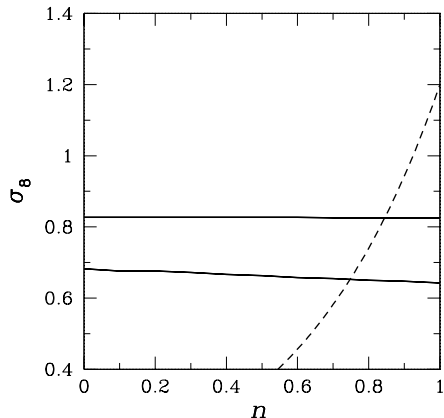Fig. 4.— Same as Figure 4, but for spatially flat CDM models.

6

Fig. 5.— Constraints on σ_8 and n for tilted CDM models. The region between the thick solid lines indicates the range allowed by the baryon fraction $f_{\rm B}$. The value of σ_8 given by the 4-year *COBE* data is shown by the thick dashed line.
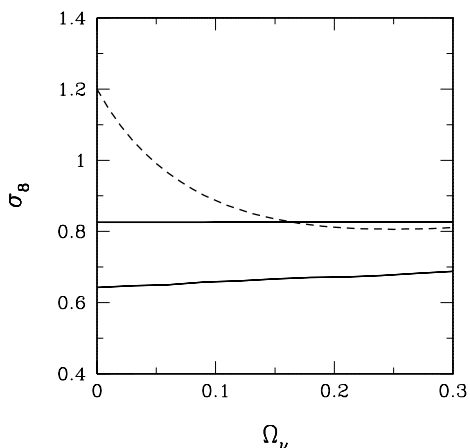

Fig. 6.— Constraints on σ_8 and Ω_ν for CHDM models. The meaning of the curves is the same as in Figure 5.

4-year *COBE* data (Bennett et al. 1996); we use the formulae given in Bunn & White (1996) to derive σ_8 for open and spatially flat CDM models. For $h = 0.5$, $\Omega_0 \simeq 0.55 - 0.65$ is required to reproduce simultaneously the baryon fraction and the *COBE* data. For $h = 0.8$, the allowed range is $\Omega_0 \simeq 0.35 - 0.45$. In any case, $\Omega_0 = 1$ models are disfavored irrespective of h. The allowed values of Ω_0 obtained here agree with those recently obtained for scale-invariant CDM models using the cluster abundance; Bahcall & Cen (1993) found that low-density ($\Omega_0 \sim 0.25 - 0.35$), low-bias ($\sigma_8 \sim 1 - 0.8$) CDM models, with or without λ_0, are consistent with the observed cluster mass function; Kitayama & Suto (1996) found that if $\Omega_0 \sim 0.2 - 0.5$ and $h = 0.7$ are taken, CDM models with the fluctuation spectrum normalized by the *COBE* data reproduce the observed cluster temperature function irrespective of λ_0; Liddle et al.(1996) concluded that for $h > 0.6$, open CDM models for which the fluctuation spectrum is normalized by the *COBE* data match the observations of galaxies, clusters, and damped Lyα systems considered in their paper only for the range $0.35 < \Omega_0 < 0.55$; Kitayama & Suto (1997) derived constraints on σ_8 and Ω_0 for CDM models from the observed $\log N$ - $\log S$ relation of X-ray clusters. As an example we plot the result obtained by Kitayama & Suto (1997) in Figure 3; the region between the dash-dotted lines indicates the allowed range obtained by Kitayama & Suto (1997) [3]. We find that for each value of h, there is an area on the Ω_0 - σ_8 plane where the model reproduces both the baryon fraction, the $\log N$ - $\log S$ relation, and the *COBE* data. The constraints on Ω_0 obtained here are also roughly consistent with those derived from the cluster baryon-to-total mass ratio and $\Omega_0^{\rm B}$ in § 4.1.

Spatially flat CDM models

Figure 4 shows the constraints on σ_8 and Ω_0 for spatially flat CDM models. The meaning of the lines is the same as in Figure 3. For $h = 0.5$, $\Omega_0 \simeq 0.35 - 0.45$ is required, while as low as $\Omega_0 \simeq 0.15 - 0.2$ is required for $h = 0.8$. These values of Ω_0 are also consistent with the previous estimates mentioned above. As an example we compare our results with that derived by Kitayama & Suto (1997) in Figure 4.

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[3]Kitayama & Suto's (1997) result plotted here is for $h = 0.7$, since they gave an analytic formula for the constraint on σ_8 and Ω_0 only for $h = 0.7$ models. However, figure 3 in their paper shows that the result is robust against varying h from 0.5 to 0.8.

For $h = 0.5$, models with $(\Omega_0, \sigma_8) \simeq (0.45, 0.8)$ match both the baryon fraction, the $\log N$ - $\log S$ relation, and the *COBE* data, though no such set of (Ω_0, σ_8) is found for $h = 0.8$.

Tilted CDM models

Figure 5 shows the results for tilted CDM models. We fix $h = 0.5$ so that the age of the universe is 13 Gyr and is consistent with that predicted from globular clusters with $(1 - 2)\sigma$ levels. The region between the two solid lines indicates the constraint from the baryon fraction. The dashed line shows σ_8 which is estimated from the 4-year *COBE* data (Ma 1996). We find that $n \simeq 0.75 - 0.85$ is required to match both the baryon fraction and the *COBE* data. Liddle et al. (1995) found that $h = 0.5$ tilted models for which the spectrum is normalized by the *COBE* data seem to be marginally consistent with the observations considered in their paper if $n \sim 0.6 - 0.7$ is taken, which is slightly smaller than the value we obtain here.

CHDM models

Finally, CHDM models are examined in Figure 6. The meaning of the lines is the same as in Figure 5. We find that $\Omega_\nu \gtrsim 0.15$ is required to reproduce the baryon fraction and the *COBE* data. This result is consistent with what Liddle et al. (1995) obtained; they found that when $h = 0.5$ and $n = 1$ are taken, $\Omega_\nu \sim 0.2 - 0.3$ models with the fluctuation spectrum normalized by the *COBE* data match the observation.

5. Conclusions

We have constructed observationally the gas mass function of clusters of galaxies to measure the fraction of baryons confined in clusters to the total baryons in the universe. Comparing this baryon fraction and the slope of the gas mass function with the prediction by the gravitational halo formation model, we have found $\sigma_8 = 0.80 \pm 0.15$ and $n \sim -1.5$ for $0.5 \leq h \leq 0.8$. Our value of σ_8 is independent of Ω_0, and thus we can estimate Ω_0 from the present result and previous ones in which σ_8 was obtained as a function of Ω_0. We have found that $\sigma_8(\Omega_0)$ derived from the cluster abundance gives $\Omega_0 \sim 0.5$ while $\sigma_8(\Omega_0)$ measured from the peculiar velocity field of galaxies gives $\Omega_0 \sim 0.2 - 1$, depending on the technique used to analyze the peculiar velocity data. We have also examined four sets of cosmic structure formation mod-

els and have found that the following models match both the observed baryon fraction and the *COBE* data; open CDM models with $\Omega_0 \simeq 0.55 - 0.65$ (for $h = 0.5$) and $\Omega_0 \simeq 0.35 - 0.45$ ($h = 0.8$); spatially flat CDM models with $\Omega_0 \simeq 0.35 - 0.45$ ($h = 0.5$) and $\Omega_0 \simeq 0.15 - 0.2$ ($h = 0.8$); tilted CDM models with $n \simeq 0.75 - 0.85$; CHDM models with $\Omega_\nu \gtrsim 0.15$.

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